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DETERMINATION OF FRACTAL DIMENSIONS OF SINGLE-VALUED SURFACES IN 3-SPACE IN THE PRESENCE OF UNIFORMLY AND NORMALLY DISTRIBUTED RANDOM NOISE: THE TRIANGULATION ALGORITHM

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13. ABSTRACT *(Maximum 200 words)*

This report describes a new and efficient algorithm for determining the Mandelbrot fractal dimension of single-valued surfaces in 3-space. First the algorithm is shown to return appropriate values for the fractal dimensions of Brown constructions. Then, in order to elucidate the "errors" introduced into measured fractal dimensions by experimental uncertainties, the algorithm is applied to Brown surfaces that have been distorted by the addition of uniformly and normally distributed "noise." Since the results apply to single-valued subsections of multiple-valued surfaces, they are also representative of the effects of uniformly and normally distributed noise on the apparent fractal scaling of multiple-valued surfaces.

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INTRODUCTION

This report describes a new fractal analysis algorithm that determines the Mandelbrot fractal dimension of surfaces in 3-space. The technique is based on analysis of a hierarchy of tessellations of a surface by triangles of varying sizes. The present implementation of the algorithm applies to single-valued functions of *xy*-plane position. Such representations of surfaces are returned, for example, by a variety of image acquisition systems. The algorithm can be applied in its present form to single-valued sections of multiple-valued surfaces and could be extended to general multiple-valued functions if a scheme (along the lines employed in FEM calculations, for instance) for defining the required polyhedral approximations (tessellations) can be devised. We refer to the present computational procedure as the "triangulation algorithm."

The usefulness of the triangulation algorithm for the analysis of exact (i.e., machine precision) surfaces is illustrated by the presentation of results of application to Brown constructions (refs 1,2), which consist of single-valued surfaces with fractal dimensions ranging from 2.0 to 3.0, and to Euclidean surfaces.

The algorithm is then employed to assess the errors introduced into measured fractal dimensions by the presence of random "noise." To accomplish this end, the triangulation algorithm is applied to measure the fractal dimensions of Brown surfaces, which have been distorted by addition of uniformly or normally distributed noise.

The algorithm has been applied to elucidate the fractal character of evolving magnetic domain wall configurations in a Barkhausen noise model system (ref 3) and of evolving structures in "Sand Pile Automata" (ref 4).

THE TRIANGULATION ALGORITHM

The triangulation algorithm employs a generalization of a technique that Mandelbrot (ref 1) refers to as "Method A" for the determination of the length of a curve in his discussion of the length of the coastline of Britain.

Mandelbrot's Method A may be thought of as a numerical approximation to an expression that describes the scaling of extremal coastline lengths

$$L(Y) = Max\left(\sum L_i\right) \tag{1a}$$

where $\{L_i \,/\, L_i \geq Y\}$ covers the curve with no overlap. The elements of the set $\{L(Y)\}$ comprise polygonal approximations to the curve under study, whose "precision" is controlled by Y, and the fractal dimension D is given by

$$L(Y) \sim Y^{1-D} \tag{1b}$$

Mandelbrot refers to the increase in the length of a coastline as the measuring scale Y is reduced in accord with Eq. (1b), as the Richardson effect (ref 5).

An analogous form for surfaces in 3-space describes the scaling of extremal tessellated areas

$$A(Y) = Max\left(\sum A_i\right) \tag{2a}$$

where $\{A_i \mid A_i \geq Y^2/2\}$ covers the surface with no overlap.

The elements of the set $\{A(Y)\}$ comprise polyhedral approximations to the surface under study, whose "precision" is controlled by Y, and the fractal dimension D is given by

$$L(Y) \sim Y^{2-D} \tag{2b}$$

Equation (2b) describes the increase of surface area as the measuring scale Y is reduced and is a manifestation of the Richardson effect.

The numerical form employed in the triangulation algorithm replaces Eq. (2a) with

$$A(Y) = \sum A_I(i, j) \tag{2c}$$

where $A_I(i, j)$ is the area of triangle I for the i, j cell on the square grid of spacing Y on the xy-plane, and $I \in \{1, 2\}$ runs over the two triangles in the i, j cell. (Triangles of fixed orientations are employed in the present implementation.) The elements of the sets $\{A_I(i, j)\}$, which together add up to $A(Y)$ for each Y comprise polyhedral approximations of the surface under study, whose "precision" is controlled by Y.

The fractal dimension is then determined from Eq. (2b)

$$D \rightarrow 2 - d(ln(A(Y))) / d(ln(Y)) \tag{2d}$$

where the "derivative" is evaluated numerically for the range of yardstick lengths Y over which fractal scaling is obtained.

One can understand the origins of Eq. (2a) by thinking in terms of the standard box-counting algorithm (ref 6). Think of a three-dimensional lattice of cubes of side Y corresponding to the tessellations based on xy-spacings Y. Then the number of "boxes" $N(Y)$ containing a piece of the polyhedral approximation at scale Y is (approximately) given by

$$N(Y) \approx A(Y)/Y^2$$

and for a fractal set of dimension D, $N(Y)$ scales as

$$N(Y) \sim Y^{-D}$$

which implies that the tessellated area at scale Y, $A(Y)$, scales as

$$A(Y) \approx Y^2 N(Y) \sim Y^{2-D}$$

and Eq. (2d) follows.

The present implementation of the triangulation algorithm requires that the section of surface under study must be a single-valued function of position on a square lattice and the yardstick lengths must be of the form

$$Y = \{2^m 3^n 5^p \ldots \mid m \in \{0, 1, \ldots, M\}, n \in \{0, 1, \ldots, N\}, \ldots\}$$

such that yardsticks in the set are consistent with covering the matrix of elevations without gaps or overlaps. It is relatively simple to generalize the algorithm to deal with rectangular lattices.

APPLICATIONS

Brown Constructions

Brown surfaces are constructed by means of a midpoint random displacement procedure as described in Reference 1. They are single-valued and can be constructed with fractal dimensions ranging from 2.0 to 3.0; thus, they constitute a suitable class of surfaces to demonstrate the usefulness of the triangulation algorithm. A level n Brown construction comprises a $(2^{(n+1)} + 1)$ by $(2^{(n+1)} + 1)$ array of elevations.

Fractal Scaling

Figure 1 shows typical loglog plots of the quantities in Eq. (2d) for level 7 (257x257) Brown constructions having $D = 2.30$ and 2.50 by construction. Linear (i.e., fractal) scaling is discernible for Y in the range of 1 to 2^4 (= 16). This range (or a larger range) of fractal scaling was observed for the Brown constructions (at level 5 and above) described here. The least squares fit lines in the figure have slopes corresponding to $D \approx 2.32$ and 2.51.

Variation in Triangulation Measured D for "Exact" Brown Constructions

The distribution of "measured" fractal dimensions determined from 500 independently constructed Brown surfaces having $D = 2.30$ by construction for levels 5 through 8 is

- $D = 2.336 \pm 0.041$ for level 5
- $D = 2.321 \pm 0.023$ for level 6
- $D = 2.313 \pm 0.015$ for level 7
- $D = 2.307 \pm 0.008$ for level 8

The fractal dimensions are expressed in the form

$$D = \langle D \rangle \pm \sigma(D)$$

where $\langle D \rangle$ is the mean of the distribution of D-values and $\sigma(D)$ is one standard deviation. Individual D-values were obtained by least squares fitting to $\log_2(A)$ versus $\log_2(Y)$ values as in Eq. (2d) for $Y = 1, 2, 4, 8$, and 16. The range of Y values was chosen because fractal scaling generally extended over at least four doublings for the Brown surfaces as discussed above.

The standard deviations $\sigma(D)$ reflect true variations in the fractal scaling of the Brown constructions for the most part. The scatter in the D values as measured by $\sigma(D)$ decreases as the level of the constructions increases, and the measured values of $\langle D \rangle$ are near to the value expected in the Brown construction algorithm. The values of $\langle D \rangle$ and $\sigma(D)$ listed above were obtained by analysis of Brown constructions initiated on unit squares having random elevations uniformly distributed between 0 and 1 at their corners; essentially the same results were obtained for Brown constructions built on flat squares.

Determination of Fractal Dimensions of Imprecisely Defined (Noisy) Surfaces

We express elevations and noise in units of xy lattice spacings, which we refer to as "pixels" to emphasize our view that important findings may result from analysis of experimental data obtained by means of pixel-based image acquisition systems.

To clarify this usage, if one acquires an image on a square lattice with spacings of 2 μm, then 1 pixel would correspond to 2 μm; and if elevations were expressed in gray levels such that one gray level corresponded to 1 μm, then one gray level would correspond to 0.5 pixel. Before application of the triangulation algorithm xy-spacings, elevations, etc. are expressed in pixels.

The measured fractal dimension of *all* the surfaces studied *increased* when either normally or uniformly distributed random values of "magnitude" greater than 0.5 pixel were added to the elevation values, and the "error" introduced in the measured fractal dimension was larger for surfaces having smaller intrinsic D values.

Figures 2 through 4 present results obtained by analyzing noisy Brown surfaces having D in the range of 2.2 to 2.8. The points on the curves were obtained by determining the statistical properties of 16 different surfaces produced by adding normally or uniformly distributed noise to the same Brown construction for each intrinsic D. Each figure shows values (connected by straight-line segments) of the mean, the mean plus, and the mean minus one standard deviation of the *increase* of the measured fractal dimension (i.e., $<\delta(D)>$, $<\delta(D)> - \sigma(\delta(D))$, and $<\delta(D)> + \sigma(\delta(D))$) for level 3 (17x17) and level 6 (129x129) noisy Brown surfaces for intrinsic D-values between 2.2 and 2.8. (The $<\delta(D)>$-values are indicated by o's.) It is interesting that the distributions of δD are relatively tight especially for the 129x129 constructions, as well as the 17x17 constructions. Since the results apply directly to single-valued subsections of multiple-valued surfaces, they are believed to be representative of the effects of noise on the apparent fractal scaling of arbitrary surfaces in 3-space.

Figure 2 presents statistical results obtained from surfaces produced by adding normally distributed noise having $\sigma = 1$ pixel to Brown constructions. The mean value for the increase in the measured D, $<\delta(D)>$, is "small" even for analysis based on 17x17 sections. The present results suggest that there is greater than an 85 percent chance that $\delta(D)$ is positive and less than 0.05 for intrinsic $D \geq 2.2$ and $\sigma = 1$-pixel normally distributed noise on 17x17 sections; a remarkable result. Thus, for example, one might obtain useful D values by applying triangulation to selections of small, relatively precise, single-valued sections of multiple-valued surfaces, etc. One can also see in Figure 2 that $<\delta(D)> + \sigma(\delta(D)) < 0.01$ for normally distributed 1-pixel noise on 129x129 surfaces having intrinsic $D > 2.25$.

Figure 3 presents statistical results similar to those of Figure 2 for surfaces produced by adding normally distributed noise having $\sigma = 3$ pixels. $<\delta(D)>$ introduced by normally distributed $\sigma = 3$-pixel noise is substantially larger than that produced by 1-pixel noise. Analysis based on 17x17 sections having $D < 2.35$ might have measured D values increased by more than 0.15. For normally distributed 3-pixel noise on 129x129 surfaces having $D > 2.2$, $<\delta(D)> + \sigma(\delta(D)) < 0.10$ and for surfaces having $D > 2.3$, $<\delta(D)> + \sigma(\delta(D)) < 0.05$. Such "errors" might be acceptable in many cases. Of course, if one knew that the measured elevations were subject to 3-pixel noise, one might be able to correct the measured D-values, etc.

Figure 4 presents statistical results similar to those of Figures 2 and 3 for surfaces produced by adding *uniformly* distributed noise having an amplitude of 3 pixels. The effect of adding uniformly distributed 3-pixel noise is, of course, much smaller than that obtained by adding normally distributed $\sigma = 3$-pixel noise. Analysis based on 17x17 constructions having $D > 2.2$, yields $<\delta(D)> + \sigma(\delta(D)) < 0.10$; analysis based on 129x129 constructions having $D > 2.2$, yields $<\delta(D)> + \sigma(\delta(D)) < 0.04$; and analysis based on 129x129 constructions having $D > 2.3$, yields $<\delta(D)> + \sigma(\delta(D)) < 0.02$.

The triangulation algorithm was also applied to determine the effects of adding uniformly and normally distributed noise to a number of magnetic domain wall model surfaces (ref 3). Increases in the apparent D were in accord with expectations based on the above-described results for Brown surfaces having the appropriate intrinsic D.

Triangulation Values of D for Single-Valued Euclidean Surfaces

The "fractal dimensions" of single-valued subsections of a number of planar surfaces, ellipsoids, and hyperboloids were determined by use of the triangulation algorithm. In all cases, the measured fractal dimension (in the $Y \to 0$ limit) was approximately two. For example, application of the triangulation algorithm to the "surface"

$$z(i,j) = (i/12.5)^2 + (j/33)^2 \text{ for } i \in 1, 2, 3, \cdots, 33 \text{ and } j \in 1, 2, 3, \cdots, 33$$

for $Y = \{1, 2, 4\}$ yields $D = 2.0005$.

SUMMARY

A new and efficient algorithm, the triangulation algorithm, for determining the Mandelbrot fractal dimension of single-valued surfaces in 3-space is described.

The effectiveness of the triangulation algorithm is remarkable. Based upon standard box-counting fractal analysis of 257x257 Brown constructions, Huang et al.(ref 7) concluded that, although measured D-values varied monotonically with the true fractal dimension, quantitative fractal analysis of surfaces was not possible. Furthermore, convergence plots for box-counting fractal analysis in Reference 6 suggest that roughly 10^5 points are required for fractal analysis of objects having $D \approx 2.0$. A more extensive study employing standard box-counting and correlation integral fractal analysis by Meisel and Johnson (ref 8) confirmed that more than 10^5 points were required to obtain convergent results for $D \approx 2.0$ and also found that about 10^6 points were required to obtain convergent results for $D \approx 2.5$.

On the other hand, the triangulation algorithm returned appropriate values for the fractal dimensions of Brown constructions for levels ranging from 4 (33x33 constructions) through 9 (1025x1025 constructions) and having fractal dimensions in the range of 2.2 to 2.8. The breadth of the distributions of D-values obtained by analysis of independently constructed Brown surfaces, reflects variations in fractal scaling from one Brown construction for a given set of parameters to another based on the same parameters.

The "errors" δD introduced into measured fractal dimensions by the presence of uniformly and normally distributed "noise" on Brown surfaces having intrinsic D-values in the range of 2.2 to 2.8 are also described. The measured D-values were larger than the intrinsic D (i.e., $\delta D > 0$) for normally distributed errors in elevations having $\sigma > 0.5$ pixel and for errors uniformly distributed over ranges greater than 0.5 pixel. Values of the mean change in D, $\langle\delta D\rangle$, and of the standard deviation of the distributions of the changes in D, $\sigma(\delta D)$, for specific normally and

uniformly distributed noise on Brown surfaces, which serve as guidelines for application of the triangulation algorithm to the analysis of noisy single-valued sections of fractal surfaces in 3-space, are presented. Since the present results apply to single-valued subsections of multiple-valued surfaces, they are believed to represent the effects of uniformly and normally distributed noise on the apparent fractal scaling of multiple-valued surfaces as well.

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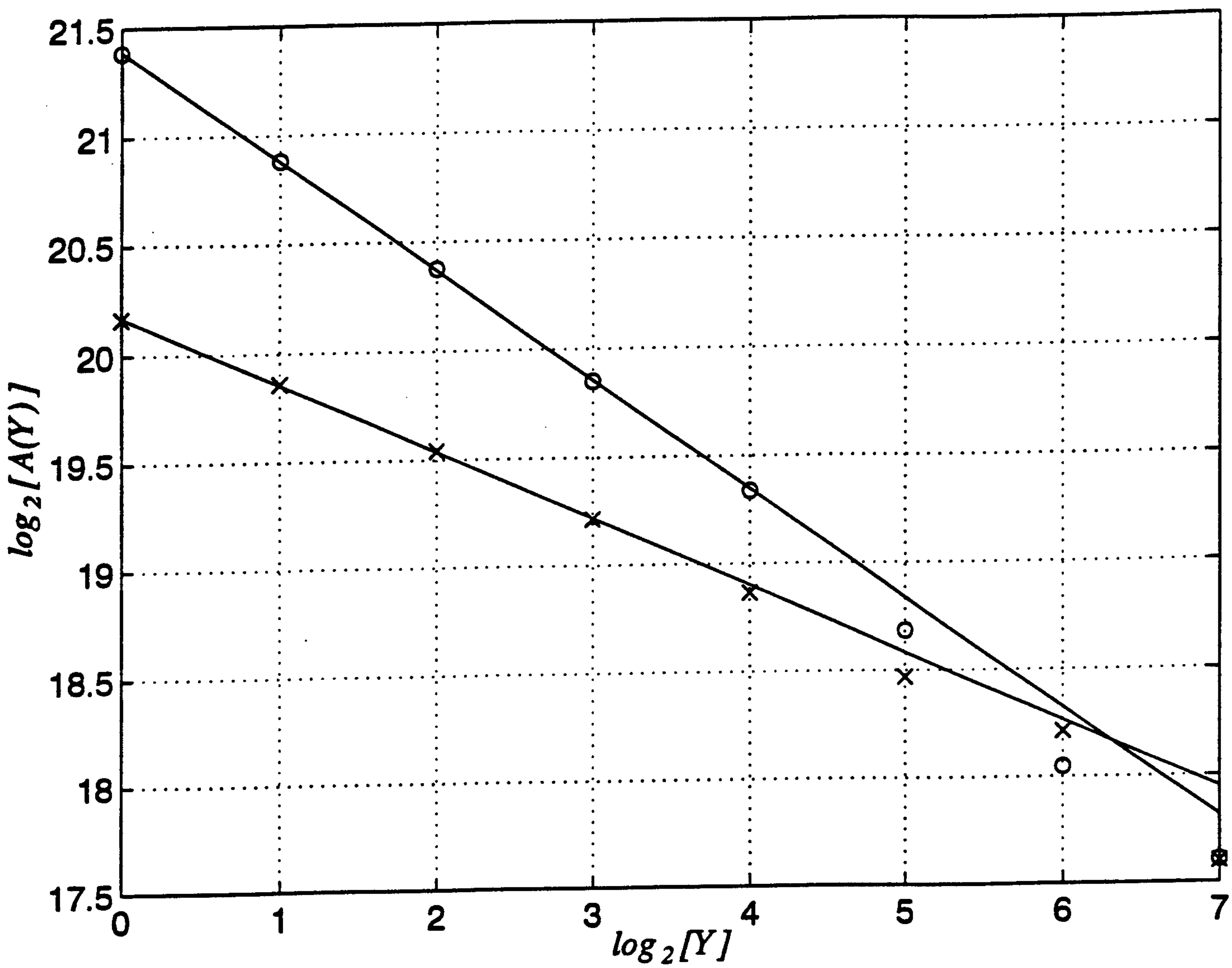

Figure 1. Typical tessellated area versus yardstick graphs for level 7 Brown surfaces. The straight lines are fit through the points at $Y = 1, 2, 4, 8$, and 16. The lines are consistent with $D = 2.32$ (and 2.51) and the data were generated from the Brown construction algorithm with parameters selected to yield $D = 2.30$ (and 2.50), respectively.

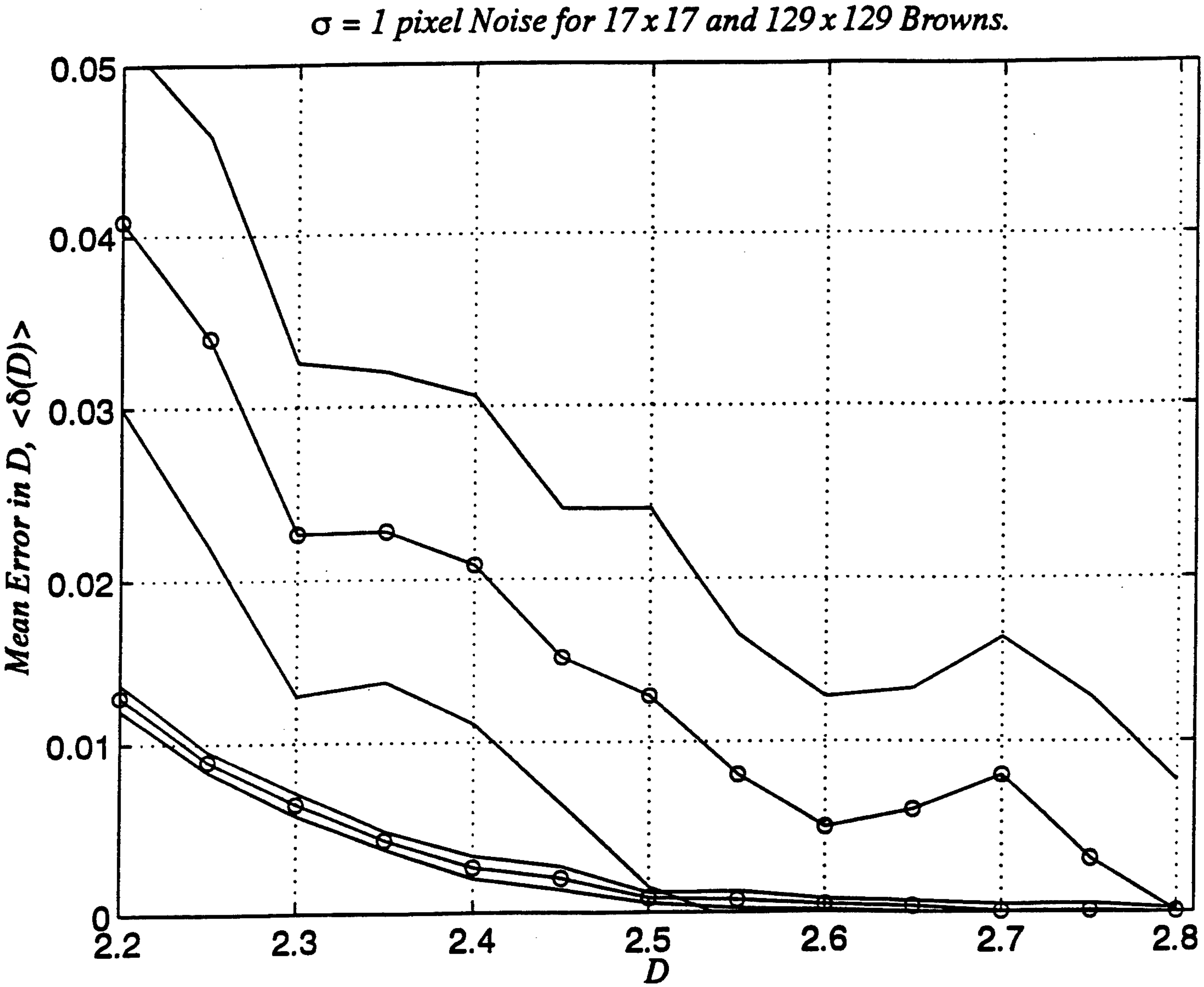

Figure 2. The mean error in D, $<\delta(D)>$, engendered by normally distributed noise having $\sigma = 1$ pixel for 17x17 and 129x129 Brown constructions having intrinsic D-values ranging from 2.2 to 2.8. In Figures (2) through (4), the outrigger lines are at $<\delta(D)> \pm \sigma(\delta(D))$ and the larger values of $<\delta(D)>$ are associated with the 17x17 constructions.

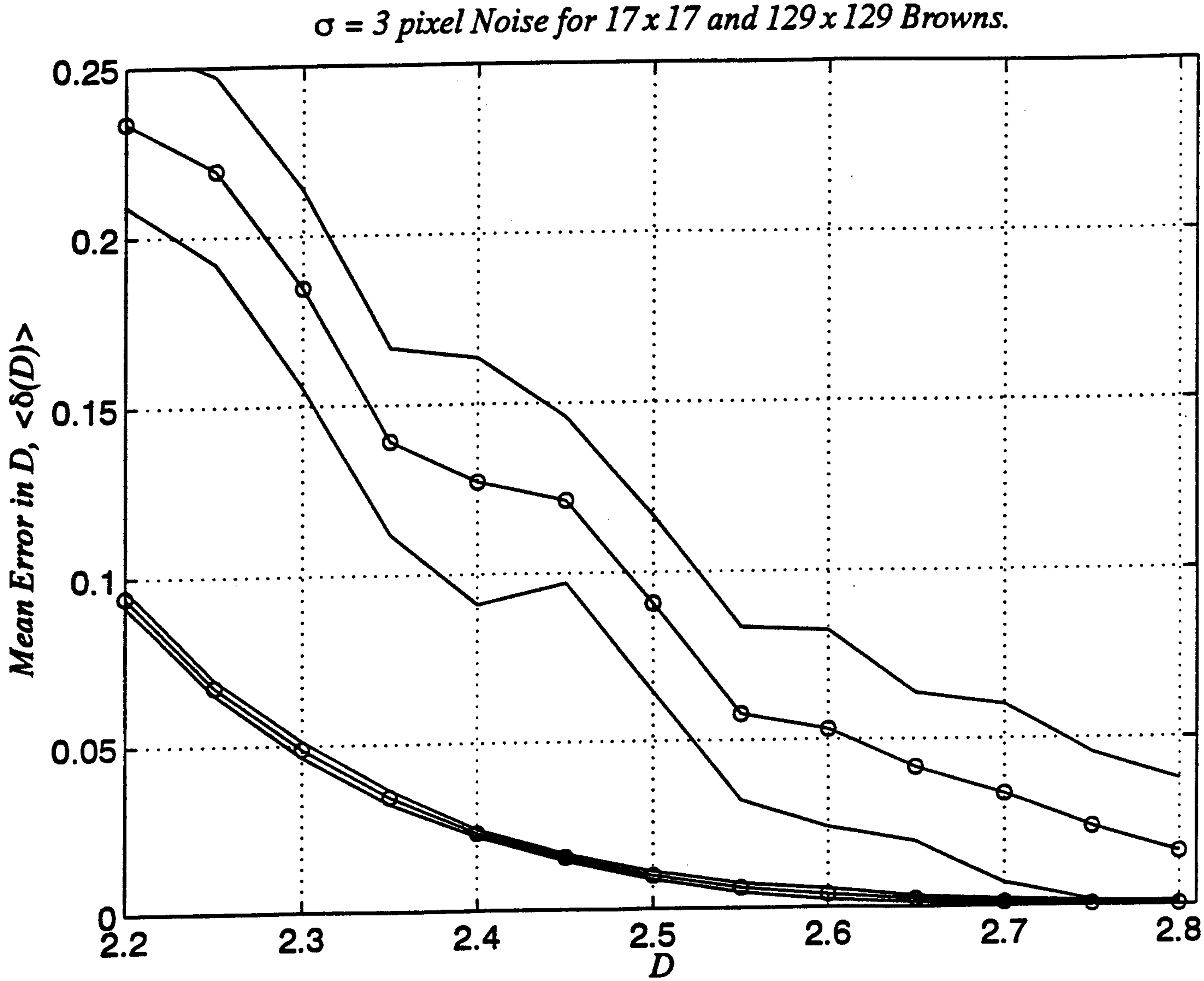

Figure 3. The mean error in D, $<\delta(D)>$, engendered by normally distributed noise having $\sigma = 3$ pixels for 17x17 and 129x129 Brown constructions having intrinsic D-values ranging from 2.2 to 2.8.

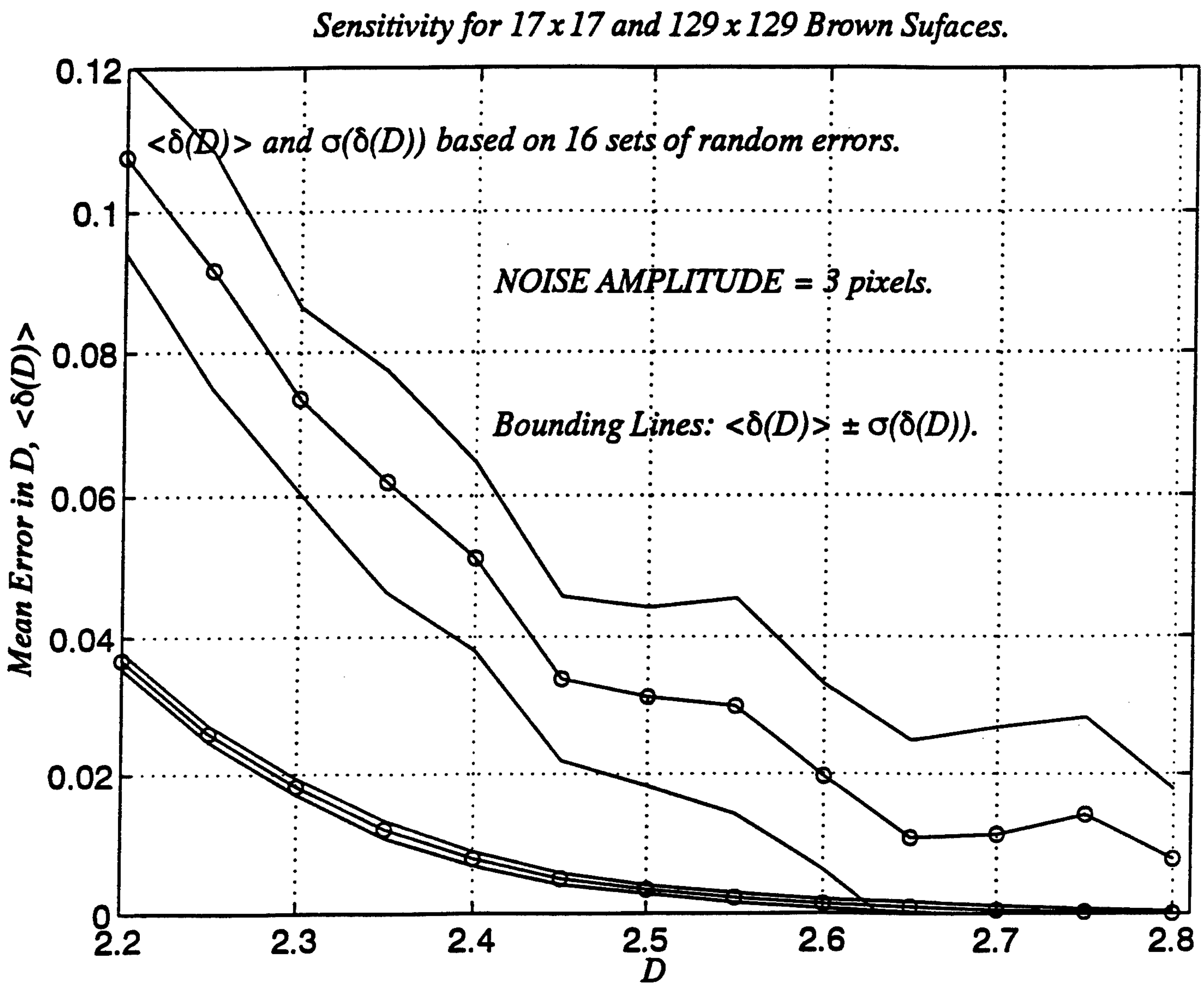

Figure 4. The mean error in D, $<\delta(D)>$, engendered by uniformly distributed noise having 3-pixel amplitude for 17x17 and 129x129 Brown constructions having intrinsic D-values ranging from 2.2 to 2.8.

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